

USING SUBSTITUTION TO FIND THE POINT OF INTERSECTION OF TWO LINES

#11

To find where two lines intersect we could graph them, but there is a faster, more accurate algebraic method called the **substitution method**. This method may also be used to solve systems of equations in word problems.

Example 1

Start with two linear equations in y-form.

$$y = -2x + 5 \quad \text{and} \quad y = x - 1$$

Substitute the equal parts.

$$-2x + 5 = x - 1$$

Solve for x.

$$6 = 3x \quad \Rightarrow \quad x = 2$$

The x-coordinate of the point of intersection is $x = 2$. To find the y-coordinate, substitute the value of x into either original equation. Solve for y , then write the solution as an ordered pair. Check that the point works in both equations.

$y = -2(2) + 5 = 1$ and $y = 2 - 1 = 1$, so $(2, 1)$ is where the lines intersect.

Check: $1 = -2(2) + 5 \quad \checkmark$ and $1 = 2 - 1 \quad \checkmark$.

Example 2

The sales of Gizmo Sports Drink at the local supermarket are currently 6,500 bottles per month. Since New Age Refreshers were introduced, sales of Gizmo have been declining by 55 bottles per month. New Age currently sells 2,200 bottles per month and its sales are increasing by 250 bottles per month. If these rates of change remain the same, in about how many months will the sales for both companies be the same? How many bottles will each company be selling at that time?

Let x = months from now and y = total monthly sales.

For Gizmo: $y = 6500 - 55x$; for New Age: $y = 2200 + 250x$.

Substituting equal parts: $6500 - 55x = 2200 + 250x \quad \Rightarrow \quad 3300 = 305x \quad \Rightarrow \quad 10.82 \approx x$.

Use either equation to find y : $y = 2200 + 250(10.82) \approx 4905$ and $y = 6500 - 55(10.82) \approx 4905$.

The solution is $(10.82, 4905)$. This means that in about 11 months, both drink companies will be selling 4,905 bottles of the sports drinks.

Find the point of intersection (x, y) for each pair of lines by using the substitution method.

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|---|--|-----------------------------------|---------------------------------|
| 1. $y = x + 2$
$y = 2x - 1$ | 2. $y = 3x + 5$
$y = 4x + 8$ | 3. $y = 11 - 2x$
$y = x + 2$ | 4. $y = 3 - 2x$
$y = 1 + 2x$ |
| 5. $y = 3x - 4$
$y = \frac{1}{2}x + 7$ | 6. $y = -\frac{2}{3}x + 4$
$y = \frac{1}{3}x - 2$ | 7. $y = 4.5 - x$
$y = -2x + 6$ | 8. $y = 4x$
$y = x + 1$ |

For each problem, define your variables, write a system of equations, and solve them by using substitution.

- Janelle has \$20 and is saving \$6 per week. April has \$150 and is spending \$4 per week. When will they both have the same amount of money?
- Sam and Hector are gaining weight for football season. Sam weighs 205 pounds and is gaining two pounds per week. Hector weighs 195 pounds but is gaining three pounds per week. In how many weeks will they both weigh the same amount?
- PhotosFast charges a fee of \$2.50 plus \$0.05 for each picture developed. PhotosQuick charges a fee of \$3.70 plus \$0.03 for each picture developed. For how many pictures will the total cost be the same at each shop?
- Playland Park charges \$7 admission plus 75¢ per ride. Funland Park charges \$12.50 admission plus 50¢ per ride. For what number of rides is the total cost the same at both parks?

Change one or both equations to y-form and solve by the substitution method.

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|----------------------------------|----------------------------------|----------------------------------|-------------------------------------|
| 13. $y = 2x - 3$
$x + y = 15$ | 14. $y = 3x + 11$
$x + y = 3$ | 15. $x + y = 5$
$2y - x = -2$ | 16. $x + 2y = 10$
$3x - 2y = -2$ |
| 17. $x + y = 3$
$2x - y = -9$ | 18. $y = 2x - 3$
$x - y = -4$ | 19. $x + 2y = 4$
$x + 2y = 6$ | 20. $3x = y - 2$
$6x + 4 = 2y$ |

Answers

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|-------------------|-----------------------------|-------------------------|--|
| 1. (3, 5) | 2. (-3, -4) | 3. (3, 5) | 4. $\left(\frac{1}{2}, 2\right)$ |
| 5. (4.4, 9.2) | 6. (6, 0) | 7. (1.5, 3) | 8. $\left(\frac{1}{3}, \frac{4}{3}\right)$ |
| 9. 13 weeks, \$98 | 10. 10 weeks,
225 pounds | 11. 60 pictures, \$5.50 | 12. 22 rides, \$23.50 |
| 13. (6, 9) | 14. (-2, 5) | 15. (4, 1) | 16. (2, 4) |
| 17. (-2, 5) | 18. (7, 11) | 19. none | 20. infinite |