

WRITING AND GRAPHING LINEAR EQUATIONS ON A FLAT SURFACE #13

SLOPE is a number that indicates the steepness (or flatness) of a line, as well as its direction (up or down) left to right.

SLOPE is determined by the ratio: $\frac{\text{vertical change}}{\text{horizontal change}}$ between any two points on a line.

For lines that go **up** (from left to right), the sign of the slope is **positive**. For lines that go **down** (left to right), the sign of the slope is **negative**.

Any linear equation written as $y = mx + b$, where m and b are any real numbers, is said to be in **SLOPE-INTERCEPT FORM**. m is the **SLOPE** of the line. b is the **Y-INTERCEPT**, that is, the point $(0, b)$ where the line intersects (crosses) the y -axis.

If two lines have the same slope, then they are parallel. Likewise, **PARALLEL LINES** have the same slope.

Two lines are **PERPENDICULAR** if the slope of one line is the negative reciprocal of the slope of the other line, that is, m and $-\frac{1}{m}$. Note that $m \cdot \left(-\frac{1}{m}\right) = -1$.

Examples: 3 and $-\frac{1}{3}$, $-\frac{2}{3}$ and $\frac{3}{2}$, $\frac{5}{4}$ and $-\frac{4}{5}$

Two distinct lines that are not parallel intersect in a single point. See "Solving Linear Systems" to review how to find the point of intersection.

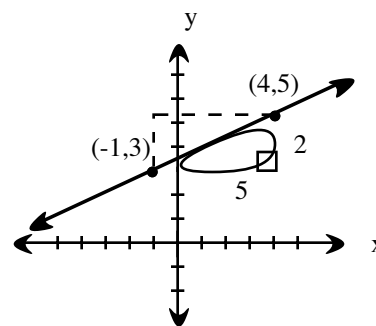
Example 1

Write the slope of the line containing the points $(-1, 3)$ and $(4, 5)$.

First graph the two points and draw the line through them.

Look for and draw a slope triangle using the two given points.

Write the ratio $\frac{\text{vertical change in } y}{\text{horizontal change in } x}$ using the legs of the right triangle: $\frac{2}{5}$.



Assign a positive or negative value to the slope (this one is positive) depending on whether the line goes up (+) or down (-) from left to right.

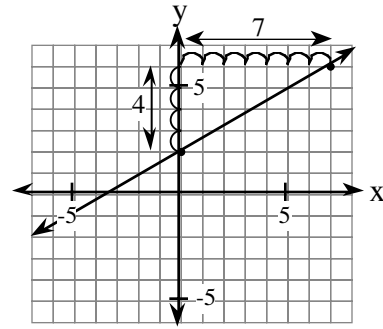
If the points are inconvenient to graph, use a "Generic Slope Triangle", visualizing where the points lie with respect to each other.

Example 2

Graph the linear equation $y = \frac{4}{7}x + 2$

Using $y = mx + b$, the slope in $y = \frac{4}{7}x + 2$ is $\frac{4}{7}$ and the y-intercept is the point $(0, 2)$. To graph, begin at the

y-intercept $(0, 2)$. Remember that slope is $\frac{\text{vertical change}}{\text{horizontal change}}$ so go up 4 units (since 4 is positive) from $(0, 2)$ and then move right 7 units. This gives a second point on the graph. To create the graph, draw a straight line through the two points.



Example 3

A line has a slope of $\frac{3}{4}$ and passes through $(3, 2)$. What is the equation of the line?

Using $y = mx + b$, write $y = \frac{3}{4}x + b$. Since $(3, 2)$ represents a point (x, y) on the line, substitute 3 for x and 2 for y , $2 = \frac{3}{4}(3) + b$, and solve for b . $2 = \frac{9}{4} + b \Rightarrow 2 - \frac{9}{4} = b \Rightarrow -\frac{1}{4} = b$. The equation is $y = \frac{3}{4}x - \frac{1}{4}$.

Example 4

Decide whether the two lines at right are parallel, perpendicular, or neither (i.e., intersecting).

$$5x - 4y = -6 \text{ and } -4x + 5y = 3.$$

First find the slope of each equation. Then compare the slopes.

$5x - 4y = -6$ $-4y = -5x - 6$ $y = \frac{-5x - 6}{-4}$ $y = \frac{5}{4}x + \frac{3}{2}$ <p>The slope of this line is $\frac{5}{4}$.</p>	$-4x + 5y = 3$ $5y = 4x + 3$ $y = \frac{4x + 3}{5}$ $y = \frac{4}{5}x + \frac{3}{5}$ <p>The slope of this line is $\frac{4}{5}$.</p>	<p>These two slopes are not equal, so they are not parallel. The product of the two slopes is 1, not -1, so they are not perpendicular. These two lines are neither parallel nor perpendicular, but do intersect.</p>
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Example 5

Find two equations of the line through the given point, one parallel and one perpendicular to the given line: $y = -\frac{5}{2}x + 5$ and $(-4, 5)$.

<p>For the parallel line, use $y = mx + b$ with the same slope to write $y = -\frac{5}{2}x + b$.</p> <p>Substitute the point $(-4, 5)$ for x and y and solve for b.</p> $5 = -\frac{5}{2}(-4) + b \Rightarrow 5 = \frac{20}{2} + b \Rightarrow -5 = b$ <p>Therefore the parallel line through $(-4, 5)$ is</p> $y = -\frac{5}{2}x - 5.$	<p>For the perpendicular line, use $y = mx + b$ where m is the negative reciprocal of the slope of the original equation to write $y = \frac{2}{5}x + b$.</p> <p>Substitute the point $(-4, 5)$ and solve for b.</p> $5 = \frac{2}{5}(-4) + b \Rightarrow \frac{33}{5} = b$ <p>Therefore the perpendicular line through $(-4, 5)$ is $y = \frac{2}{5}x + \frac{33}{5}$.</p>
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Write the slope of the line containing each pair of points.

1. (3, 4) and (5, 7) 2. (5, 2) and (9, 4) 3. (1, -3) and (-4, 7)
 4. (-2, 1) and (2, -2) 5. (-2, 3) and (4, 3) 6. (8, 5) and (3, 5)

Use a Generic Slope Triangle to write the slope of the line containing each pair of points:

7. (51, 40) and (33, 72) 8. (20, 49) and (54, 90) 9. (10, -13) and (-61, 20)

Identify the y-intercept in each equation.

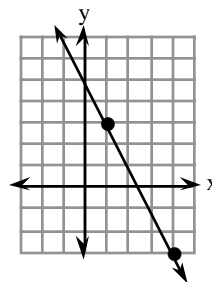
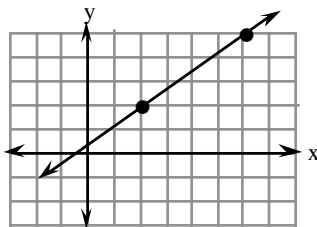
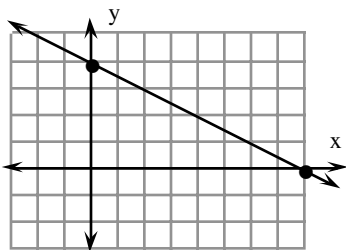
10. $y = \frac{1}{2}x - 2$ 11. $y = -\frac{3}{5}x - \frac{5}{3}$ 12. $3x + 2y = 12$
 13. $x - y = -13$ 14. $2x - 4y = 12$ 15. $4y - 2x = 12$

Write the equation of the line with:

16. slope = $\frac{1}{2}$ and passing through (4, 3). 17. slope = $\frac{2}{3}$ and passing through (-3, -2).
 18. slope = $-\frac{1}{3}$ and passing through (4, -1). 19. slope = -4 and passing through (-3, 5).

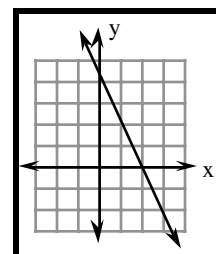
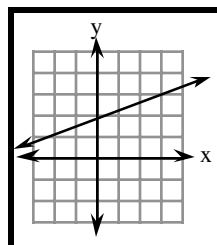
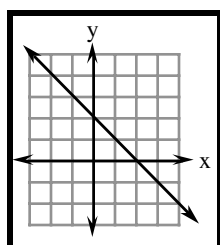
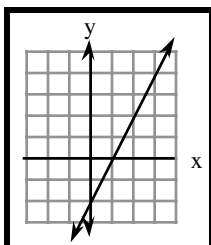
Determine the slope of each line using the highlighted points.

20. 21. 22.



Using the slope and y-intercept, determine the equation of the line.

23. 24. 25. 26.



Graph the following linear equations on graph paper.

27. $y = \frac{1}{2}x + 3$

28. $y = -\frac{3}{5}x - 1$

29. $y = 4x$

30. $y = -6x + \frac{1}{2}$

31. $3x + 2y = 12$

State whether each pair of lines is parallel, perpendicular, or intersecting.

32. $y = 2x - 2$ and $y = 2x + 4$

33. $y = \frac{1}{2}x + 3$ and $y = -2x - 4$

34. $x - y = 2$ and $x + y = 3$

35. $y - x = -1$ and $y + x = 3$

36. $x + 3y = 6$ and $y = -\frac{1}{3}x - 3$

37. $3x + 2y = 6$ and $2x + 3y = 6$

38. $4x = 5y - 3$ and $4y = 5x + 3$

39. $3x - 4y = 12$ and $4y = 3x + 7$

Find an equation of the line through the given point and parallel to the given line.

40. $y = 2x - 2$ and $(-3, 5)$

41. $y = \frac{1}{2}x + 3$ and $(-4, 2)$

42. $x - y = 2$ and $(-2, 3)$

43. $y - x = -1$ and $(-2, 1)$

44. $x + 3y = 6$ and $(-1, 1)$

45. $3x + 2y = 6$ and $(2, -1)$

46. $4x = 5y - 3$ and $(1, -1)$

47. $3x - 4y = 12$ and $(4, -2)$

Find an equation of the line through the given point and perpendicular to the given line.

48. $y = 2x - 2$ and $(-3, 5)$

49. $y = \frac{1}{2}x + 3$ and $(-4, 2)$

50. $x - y = 2$ and $(-2, 3)$

51. $y - x = -1$ and $(-2, 1)$

52. $x + 3y = 6$ and $(-1, 1)$

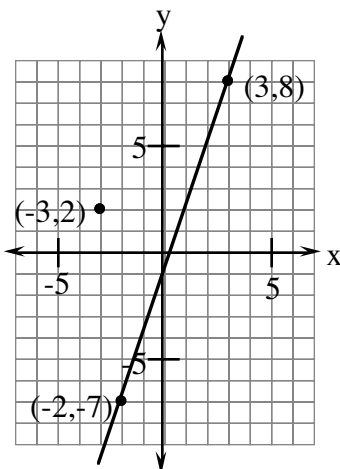
53. $3x + 2y = 6$ and $(2, -1)$

54. $4x = 5y - 3$ and $(1, -1)$

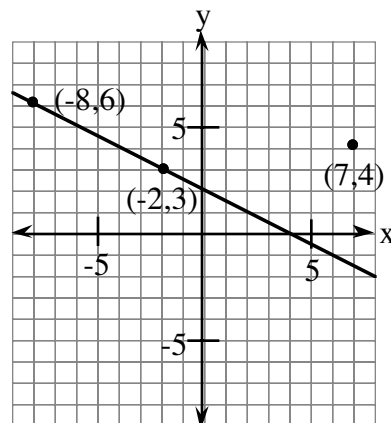
55. $3x - 4y = 12$ and $(4, -2)$

Write an equation of the line parallel to each line below through the given point.

56.



57.



Answers

1. $\frac{3}{2}$

2. $\frac{1}{2}$

3. -2

4. $-\frac{3}{4}$

5. 0

6. 0

7. $-\frac{16}{9}$

8. $\frac{41}{34}$

9. $\frac{-33}{71}$

10. $(0, -2)$

11. $(0, -\frac{5}{3})$

12. $(0, 6)$

13. $(0, 13)$

14. $(0, -3)$

15. $(0, 3)$

16. $y = \frac{1}{2}x + 1$

17. $y = \frac{2}{3}x$

18. $y = -\frac{1}{3}x + \frac{1}{3}$

19. $y = -4x - 7$

20. $-\frac{1}{2}$

21. $\frac{3}{4}$

22. -2

23. $y = 2x - 2$

24. $y = -x + 2$

25. $y = \frac{1}{3}x + 2$

26. $y = -2x + 4$

27. line with slope $\frac{1}{2}$ and y-intercept $(0, 3)$

28. line with slope $-\frac{3}{5}$ and y-intercept $(0, -1)$

29. line with slope 4 and y-intercept $(0, 0)$

30. line with slope -6 and y-intercept $(0, \frac{1}{2})$

31. line with slope $-\frac{3}{2}$ and y-intercept $(0, 6)$

32. parallel

33. perpendicular

34. perpendicular

35. perpendicular

36. parallel

37. intersecting

38. intersecting

39. parallel

40. $y = 2x + 11$

41. $y = \frac{1}{2}x + 4$

42. $y = x + 5$

43. $y = x + 3$

44. $y = -\frac{1}{3}x + \frac{2}{3}$

45. $y = -\frac{3}{2}x + 2$

46. $y = \frac{4}{5}x - \frac{9}{5}$

47. $y = \frac{3}{4}x - 5$

48. $y = -\frac{1}{2}x + \frac{7}{2}$

49. $y = -2x - 6$

50. $y = -x + 1$

51. $y = -x - 1$

52. $y = 3x + 4$

53. $y = \frac{2}{3}x - \frac{7}{3}$

54. $y = -\frac{5}{4}x + \frac{1}{4}$

55. $y = -\frac{4}{3}x + \frac{10}{3}$

56. $y = 3x + 11$

57. $y = -\frac{1}{2}x + \frac{15}{2}$