

FACTORING POLYNOMIALS

#14

Often we want to un-multiply or **factor** a polynomial $P(x)$. This process involves finding a constant and/or another polynomial that evenly divides the given polynomial. In formal mathematical terms, this means $P(x) = q(x) \cdot r(x)$, where q and r are also polynomials. For elementary algebra there are three general types of factoring.

- 1) **Common term** (finding the largest common factor):

$$6x + 18 = 6(x + 3) \text{ where } 6 \text{ is a common factor of both terms.}$$

$$2x^3 - 8x^2 - 10x = 2x(x^2 - 4x - 5) \text{ where } 2x \text{ is the common factor.}$$

$$2x^2(x - 1) + 7(x - 1) = (x - 1)(2x^2 + 7) \text{ where } x - 1 \text{ is the common factor.}$$

- 2) **Special products**

$$a^2 - b^2 = (a + b)(a - b) \quad x^2 - 25 = (x + 5)(x - 5)$$

$$9x^2 - 4y^2 = (3x + 2y)(3x - 2y)$$

$$x^2 + 2xy + y^2 = (x + y)^2 \quad x^2 + 8x + 16 = (x + 4)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2 \quad x^2 - 8x + 16 = (x - 4)^2$$

- 3a) **Trinomials** in the form $x^2 + bx + c$ where the coefficient of x^2 is 1.

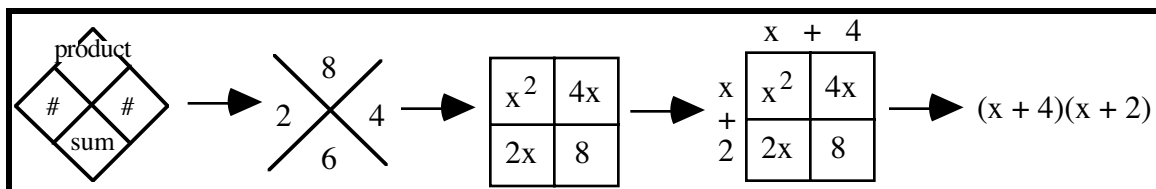
Consider $x^2 + (d + e)x + d \cdot e = (x + d)(x + e)$, where the coefficient of x is the sum of two numbers d and e AND the constant is the product of the same two numbers, d and e . A quick way to determine all of the possible pairs of integers d and e is to factor the constant in the original trinomial. For example, 12 is $1 \cdot 12$, $2 \cdot 6$, and $3 \cdot 4$. The signs of the two numbers are determined by the combination you need to get the sum. The "sum and product" approach to factoring trinomials is the same as solving a "Diamond Problem" in CPM's Algebra 1 course (see below).

$$x^2 + 8x + 15 = (x + 3)(x + 5); \quad 3 + 5 = 8, \quad 3 \cdot 5 = 15$$

$$x^2 - 2x - 15 = (x - 5)(x + 3); \quad -5 + 3 = -2, \quad -5 \cdot 3 = -15$$

$$x^2 - 7x + 12 = (x - 3)(x - 4); \quad -3 + (-4) = -7, \quad (-3)(-4) = 12$$

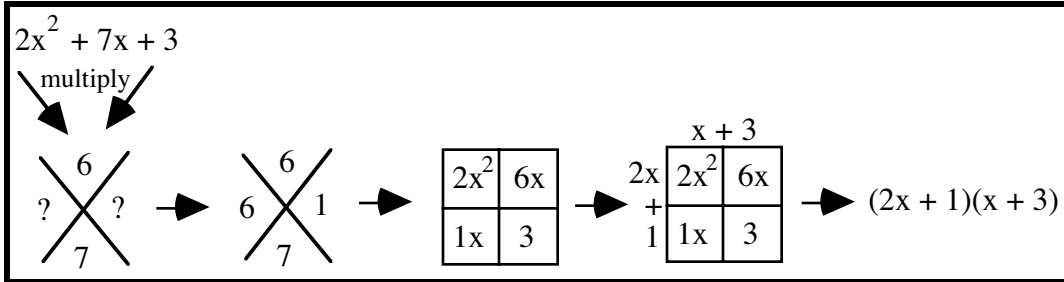
The sum and product approach can be shown visually using rectangles for an area model. The figure at far left below shows the "Diamond Problem" format for finding a sum and product. Here is how to use this method to factor $x^2 + 6x + 8$.



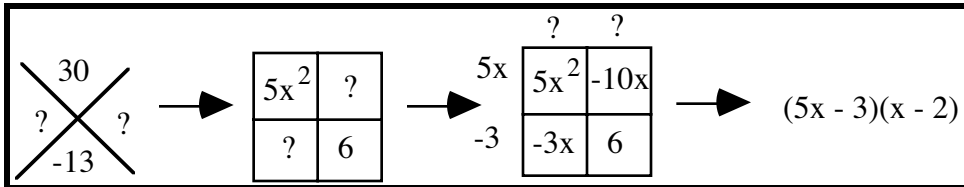
>> Explanation and examples continue on the next page. >>

3b) **Trinomials** in the form $ax^2 + bx + c$ where $a \neq 1$.

Note that the upper value in the diamond is no longer the constant. Rather, it is the product of a and c , that is, the coefficient of x^2 and the constant.



Below is the process to factor $5x^2 - 13x + 6$.



Polynomials with four or more terms are generally factored by grouping the terms and using one or more of the three procedures shown above. Note that polynomials are usually factored completely. In the second example in part (1) above, the trinomial also needs to be factored. Thus, the complete factorization of $2x^3 - 8x^2 - 10x = 2x(x^2 - 4x - 5) = 2x(x - 5)(x + 1)$.

Factor each polynomial completely.

1. $x^2 - x - 42$
2. $4x^2 - 18$
3. $2x^2 + 9x + 9$
4. $2x^2 + 3xy + y^2$
5. $6x^2 - x - 15$
6. $4x^2 - 25$
7. $x^2 - 28x + 196$
8. $7x^2 - 847$
9. $x^2 + 18x + 81$
10. $x^2 + 4x - 21$
11. $3x^2 + 21x$
12. $3x^2 - 20x - 32$
13. $9x^2 - 16$
14. $4x^2 + 20x + 25$
15. $x^2 - 5x + 6$
16. $5x^3 + 15x^2 - 20x$
17. $4x^2 + 18$
18. $x^2 - 12x + 36$
19. $x^2 - 3x - 54$
20. $6x^2 - 21$
21. $2x^2 + 15x + 18$
22. $16x^2 - 1$
23. $x^2 - 14x + 49$
24. $x^2 + 8x + 15$
25. $3x^3 - 12x^2 - 45x$
26. $3x^2 + 24$
27. $x^2 + 16x + 64$

Factor completely.

28. $75x^3 - 27x$
29. $3x^3 - 12x^2 - 36x$
30. $4x^3 - 44x^2 + 112x$
31. $5y^2 - 125$
32. $3x^2y^2 - xy^2 - 4y^2$
33. $x^3 + 10x^2 - 24x$
34. $3x^3 - 6x^2 - 45x$
35. $3x^2 - 27$
36. $x^4 - 16$

Factor each of the following completely. Use the modified diamond approach.

37. $2x^2 + 5x - 7$

38. $3x^2 - 13x + 4$

39. $2x^2 + 9x + 10$

40. $4x^2 - 13x + 3$

41. $4x^2 + 12x + 5$

42. $6x^3 + 31x^2 + 5x$

43. $64x^2 + 16x + 1$

44. $7x^2 - 33x - 10$

45. $5x^2 + 12x - 9$

Answers

1. $(x + 6)(x - 7)$

2. $2(2x^2 - 9)$

3. $(2x + 3)(x + 3)$

4. $(2x + y)(x + y)$

5. $(2x + 3)(3x - 5)$

6. $(2x - 5)(2x + 5)$

7. $(x - 14)^2$

8. $7(x - 11)(x + 11)$

9. $(x + 9)^2$

10. $(x + 7)(x - 3)$

11. $3x(x + 7)$

12. $(x - 8)(3x + 4)$

13. $(3x - 4)(3x + 4)$

14. $(2x + 5)^2$

15. $(x - 3)(x - 2)$

16. $5x(x + 4)(x - 1)$

17. $2(2x^2 + 9)$

18. $(x - 6)^2$

19. $(x - 9)(x + 6)$

20. $3(2x^2 - 7)$

21. $(2x + 3)(x + 6)$

22. $(4x + 1)(4x - 1)$

23. $(x - 7)^2$

24. $(x + 3)(x + 5)$

25. $3x(x^2 - 4x - 15)$

26. $3(x^2 + 8)$

27. $(x + 8)^2$

28. $3x(5x - 3)(5x + 3)$

29. $3x(x - 6)(x + 2)$

30. $4x(x - 7)(x - 4)$

31. $5(y + 5)(y - 5)$

32. $y^2(3x - 4)(x + 1)$

33. $x(x + 12)(x - 2)$

34. $3x(x - 5)(x + 3)$

35. $3(x - 3)(x + 3)$

36. $(x - 2)(x + 2)(x^2 + 4)$

37. $(2x + 7)(x - 1)$

38. $(3x - 1)(x - 4)$

39. $(x + 2)(2x + 5)$

40. $(4x - 1)(x - 3)$

41. $(2x + 5)(2x + 1)$

42. $x(6x + 1)(x + 5)$

43. $(8x + 1)^2$

44. $(7x + 2)(x - 5)$

45. $(5x - 3)(x + 3)$