

ZERO PRODUCT PROPERTY AND QUADRATICS

#15

If $a \cdot b = 0$, then either $a = 0$ or $b = 0$.

Note that this property states that at least one of the factors **MUST** be zero. It is also possible that all of the factors are zero. This simple statement gives us a powerful result which is most often used with equations involving the products of binomials. For example, solve $(x + 5)(x - 2) = 0$.

By the Zero Product Property, since $(x + 5)(x - 2) = 0$, either $x + 5 = 0$ or $x - 2 = 0$. Thus, $x = -5$ or $x = 2$.

The Zero Product Property can be used to find where a quadratic crosses the x-axis. These points are the x-intercepts. In the example above, they would be $(-5, 0)$ and $(2, 0)$.

Here are two more examples. Solve each quadratic equation and check each solution.

Example 1

$$(x + 4)(x - 7) = 0$$

By the Zero Product Property,
either $x + 4 = 0$ or $x - 7 = 0$
Solving, $x = -4$ or $x = 7$.

Checking,

$$(-4 + 4)(-4 - 7) \stackrel{?}{=} 0$$

$$(0)(-11) = 0 \checkmark$$

$$(7 + 4)(7 - 7) \stackrel{?}{=} 0$$

$$(11)(0) = 0 \checkmark$$

Example 2

$$x^2 + 3x - 10 = 0$$

First factor $x^2 + 3x - 10 = 0$
into $(x + 5)(x - 2) = 0$
then $x + 5 = 0$ or $x - 2 = 0$,
so $x = -5$ or $x = 2$

Checking,

$$(-5 + 5)(-5 - 2) \stackrel{?}{=} 0$$

$$(0)(-7) = 0 \checkmark$$

$$(2 + 5)(2 - 2) \stackrel{?}{=} 0$$

$$(7)(0) = 0 \checkmark$$

Solve each of the following quadratic equations.

1. $(x + 7)(x + 1) = 0$

2. $(x + 2)(x + 3) = 0$

3. $x(x - 2) = 0$

4. $x(x - 7) = 0$

5. $(3x - 3)(4x + 2) = 0$

6. $(2x + 5)(4x - 3) = 0$

7. $x^2 + 4x + 3 = 0$

8. $x^2 + 6x + 5 = 0$

9. $x^2 - 6x + 8 = 0$

10. $x^2 - 8x + 15 = 0$

11. $x^2 + x = 6$

12. $x^2 - x = 6$

13. $x^2 - 10x = -16$

14. $x^2 - 11x = -28$

Without graphing, find where each parabola crosses the x-axis.

15. $y = x^2 - 2x - 3$

16. $y = x^2 + 2x - 8$

17. $y = x^2 - x - 30$

18. $y = x^2 + 4x - 5$

19. $x^2 + 4x = 5 + y$

20. $x^2 - 3x = 10 + y$

Answers

1. $x = -7$ and $x = -1$

2. $x = -2$ and $x = -3$

3. $x = 0$ and $x = 2$

4. $x = 0$ and $x = 7$

5. $x = 1$ and $x = -\frac{1}{2}$

6. $x = \frac{-5}{2}$ and $x = \frac{3}{4}$

7. $x = -1$ and $x = -3$

8. $x = -1$ and $x = -5$

9. $x = 4$ and $x = 2$

10. $x = 5$ and $x = 3$

11. $x = -3$ and $x = 2$

12. $x = 3$ and $x = -2$

13. $x = 2$ and $x = 8$

14. $x = 4$ and $x = 7$

15. $(-1, 0)$ and $(3, 0)$

16. $(-4, 0)$ and $(2, 0)$

17. $(6, 0)$ and $(-5, 0)$

18. $(-5, 0)$ and $(1, 0)$

19. $(1, 0)$ and $(-5, 0)$

20. $(5, 0)$ and $(-2, 0)$