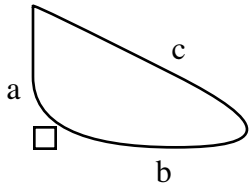


PYTHAGOREAN THEOREM

#16



Any triangle that has a right angle is called a **right triangle**. The two sides that form the right angle, a and b, are called **legs**, and the side opposite (that is, across the triangle from) the right angle, c, is called the **hypotenuse**.

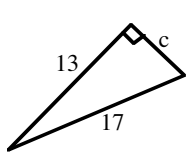
For any right triangle, the sum of the squares of the legs of the triangle is equal to the square of the hypotenuse, that is, $a^2 + b^2 = c^2$. This relationship is known as the **Pythagorean Theorem**. In words, the theorem states that:

$$(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2.$$

Example

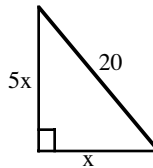
Draw a diagram, then use the Pythagorean Theorem to write an equation or use area pictures (as shown on page 22, problem RC-1) on each side of the triangle to solve each problem.

a) Solve for the missing side.



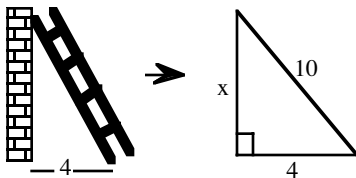
$$\begin{aligned} c^2 + 13^2 &= 17^2 \\ c^2 + 169 &= 289 \\ c^2 &= 120 \\ c &= \sqrt{120} \\ c &= 2\sqrt{30} \\ c &\approx 10.95 \end{aligned}$$

b) Find x to the nearest tenth:



$$\begin{aligned} (5x)^2 + x^2 &= 20^2 \\ 25x^2 + x^2 &= 400 \\ 26x^2 &= 400 \\ x^2 &\approx 15.4 \\ x &\approx \sqrt{15.4} \\ x &\approx 3.9 \end{aligned}$$

c) One end of a ten foot ladder is four feet from the base of a wall. How high on the wall does the top of the ladder touch?



$$\begin{aligned} x^2 + 4^2 &= 10^2 \\ x^2 + 16 &= 100 \\ x^2 &= 84 \\ x &\approx 9.2 \end{aligned}$$

d) Could 3, 6 and 8 represent the lengths of the sides of a right triangle? Explain.

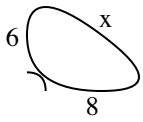
$$\begin{aligned} 3^2 + 6^2 &\stackrel{?}{=} 8^2 \\ 9 + 36 &\stackrel{?}{=} 64 \\ 45 &\neq 64 \end{aligned}$$

Since the Pythagorean Theorem relationship is not true for these lengths, they cannot be the side lengths of a right triangle.

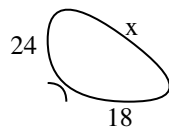
The ladder touches the wall about 9.2 feet above the ground.

Write an equation and solve for each unknown side. Round to the nearest hundredth.

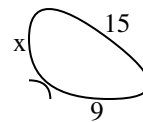
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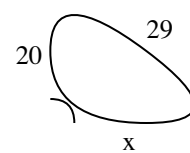
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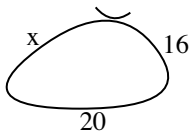
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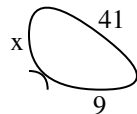
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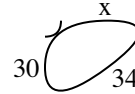
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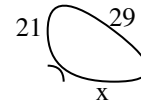
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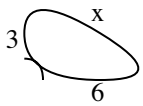
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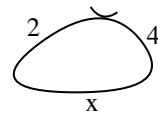
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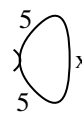
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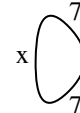
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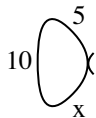
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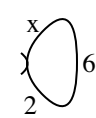
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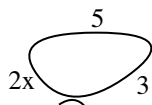


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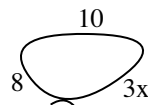


Be careful! Remember to square the whole side. For example, $(2x)^2 = (2x)(2x) = 4x^2$.

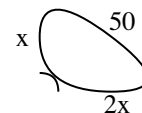
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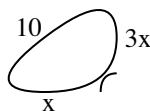
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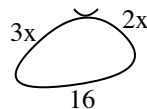
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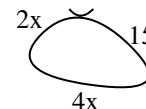
18.



19.



20.



For each of the following problems draw and label a diagram. Then write an equation using the Pythagorean Theorem and solve for the unknown. Round answers to the nearest hundredth.

21. In a right triangle, the length of the hypotenuse is four inches. The length of one leg is two inches. Find the length of the other leg.
22. The length of the hypotenuse of a right triangle is six cm. The length of one leg is four cm. Find the length of the other leg.
23. Find the diagonal length of a television screen 30 inches wide by 20 inches long.
24. Find the length of a path that runs diagonally across a 53 yard by 100 yard field.
25. A mover must put a circular mirror two meters in diameter through a one meter by 1.8 meter doorway. Find the length of the diagonal of the doorway. Will the mirror fit?

26. A surveyor walked eight miles north, then three miles west. How far was she from her starting point?
27. A four meter ladder is one meter from the base of a building. How high up the building will the ladder reach?
28. A 12-meter loading ramp rises to the edge of a warehouse doorway. The bottom of the ramp is nine meters from the base of the warehouse wall. How high above the base of the wall is the doorway?
29. What is the longest line you can draw on a paper that is 15 cm by 25 cm?
30. How long an umbrella will fit in the bottom of a suitcase that is 2.5 feet by 3 feet?
31. How long a guy wire is needed to support a 10 meter tall tower if it is fastened five meters from the foot of the tower?
32. Find the diagonal distance from one corner of a 30 foot square classroom floor to the other corner of the floor.
33. Harry drove 10 miles west, then five miles north, then three miles west. How far was he from his starting point?
34. Linda can turn off her car alarm from 20 yards away. Will she be able to do it from the far corner of a 15 yard by 12 yard parking lot?
35. The hypotenuse of a right triangle is twice as long as one of its legs. The other leg is nine inches long. Find the length of the hypotenuse.
36. One leg of a right triangle is three times as long as the other. The hypotenuse is 100 cm. Find the length of the shorter leg.

Answers

- | | | | |
|--|----------------------|--|----------------------|
| 1. $x = 10$ | 2. $x = 30$ | 3. $x = 12$ | 4. $x = 21$ |
| 5. $x = 12$ | 6. $x = 40$ | 7. $x = 16$ | 8. $x = 20$ |
| 9. $x \approx 6.71$ | 10. $x \approx 4.47$ | 11. $x \approx 7.07$ | 12. $x \approx 9.9$ |
| 13. $x \approx 8.66$ | 14. $x \approx 5.66$ | 15. $x = 2$ | 16. $x = 2$ |
| 17. $x \approx 22.36$ | 18. $x \approx 3.16$ | 19. $x \approx 4.44$ | 20. $x \approx 4.33$ |
| 21. 3.46 inches | 22. 4.47 cm | 23. 36.06 inches | 24. 113.18 yards |
| 25. The diagonal is 2.06 meters, so yes. | 26. 8.54 miles | 27. 3.87 meters | |
| 28. 7.94 meters | 29. 29.15 cm | 30. 3.91 feet | 31. 11.18 meters |
| 32. 42.43 feet | 33. 13.93 miles | 34. The corner is 19.21 yards away so yes! | |
| 35. 10.39 inches | 36. 31.62 cm | | |