

LAWS OF EXPONENTS

#18

BASE, EXPONENT, AND VALUE

In the expression 2^5 , 2 is the **base**, 5 is the **exponent**, and the **value** is 32.

$$2^5 \text{ means } 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$$

$$x^3 \text{ means } x \cdot x \cdot x$$

LAWS OF EXPONENTS

Here are the basic patterns with examples:

$$1) \quad x^a \cdot x^b = x^{a+b}$$

$$\text{examples: } x^3 \cdot x^4 = x^{3+4} = x^7;$$

$$2^7 \cdot 2^4 = 2^{11}$$

$$2) \quad \frac{x^a}{x^b} = x^{a-b}$$

$$\text{examples: } x^{10} \div x^4 = x^{10-4} = x^6;$$

$$\frac{2^4}{2^7} = 2^{-3} \text{ or } \frac{1}{2^3}$$

$$3) \quad (x^a)^b = x^{ab}$$

$$\text{examples: } (x^4)^3 = x^{4 \cdot 3} = x^{12};$$

$$(2x^3)^4 = 2^4 \cdot x^{12} = 16x^{12}$$

$$4) \quad x^{-a} = \frac{1}{x^a} \text{ and } \frac{1}{x^{-b}} = x^b$$

$$\text{examples: } 3x^{-3}y^2 = \frac{3y^2}{x^3};$$

$$\frac{2x^5}{y^{-2}} = 2x^5y^2$$

$$5) \quad x^0 = 1$$

$$\text{examples: } 5^0 = 1;$$

$$(2x)^0 = 1$$

Example 1

$$\text{Simplify: } (2xy^3)(5x^2y^4)$$

$$\text{Multiply the coefficients: } 2 \cdot 5 \cdot xy^3 \cdot x^2y^4 = 10xy^3 \cdot x^2y^4$$

$$\text{Add the exponents of } x, \text{ then } y: 10x^{1+2}y^{3+4} = 10x^3y^7$$

Example 2

$$\text{Simplify: } \frac{14x^2y^{12}}{7x^5y^7}$$

$$\text{Divide the coefficients: } \frac{(14 \div 7)x^2y^{12}}{x^5y^7} = \frac{2x^2y^{12}}{x^5y^7}$$

$$\text{Subtract the exponents: } 2x^{2-5}y^{12-7} = 2x^{-3}y^5 \text{ OR } \frac{2y^5}{x^3}$$

Example 3

$$\text{Simplify: } (3x^2y^4)^3$$

$$\text{Cube each factor: } 3^3 \cdot (x^2)^3 \cdot (y^4)^3 = 27(x^2)^3(y^4)^3$$

$$\text{Multiply the exponents: } 27x^6y^{12}$$

Example 4

Simplify: $3x^{-4}y^2z^{-3} \Rightarrow \frac{3y^2}{x^4z^3}$

Simplify each expression:

1. $y^5 \cdot y^7$
2. $b^4 \cdot b^3 \cdot b^2$
3. $8^6 \cdot 8^2$
4. $(y^5)^2$
5. $(3a)^4$
6. $\frac{m^8}{m^3}$
7. $\frac{12x^9}{4x^4}$
8. $(x^3y^2)^3$
9. $\frac{(y^4)^2}{(y^3)^2}$
10. $\frac{15x^2y^7}{3x^4y^5}$
11. $(4c^4)(ac^3)(3a^5c)$
12. $(7x^3y^5)^2$
13. $(4xy^2)(2y)^3$
14. $\left(\frac{4}{x^2}\right)^3$
15. $\frac{(2a^7)(3a^2)}{6a^3}$
16. $\left(\frac{5m^3n}{m^5}\right)^3$
17. $(3a^2x^3)^2(2ax^4)^3$
18. $\left(\frac{x^3y}{y^4}\right)^4$
19. $\left(\frac{6y^2x^8}{12x^3y^7}\right)^2$
20. $\frac{(2x^5y^3)^3(4xy^4)^2}{8x^7y^{12}}$

Write the following expressions without negative exponents.

21. x^{-2}
22. $y^{-3}y^2$
23. $\frac{x}{x^{-2}}$
24. $(y^{-2})^3$

Note: More practice with negative exponents is available in Skill Builder #21.

Answers

1. y^{12}
2. b^9
3. 8^8
4. y^{10}
5. $81a^4$
6. m^5
7. $3x^5$
8. x^9y^6
9. y^2
10. $\frac{5y^2}{x^2}$
11. $12a^6c^8$
12. $49x^6y^{10}$
13. $32xy^5$
14. $\frac{64}{x^6}$
15. a^6
16. $\frac{125n^3}{m^6}$
17. $72a^7x^{18}$
18. $\frac{x^{12}}{y^{12}}$
19. $\frac{x^{10}}{4y^{10}}$
20. $16x^{10}y^5$
21. $\frac{1}{x^2}$
22. $\frac{1}{y}$
23. x^7
24. $\frac{1}{y^{10}}$