

## THE QUADRATIC FORMULA

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You have used factoring and the Zero Product Property to solve quadratic equations. You can solve any quadratic equation by using the **quadratic formula**.

$$\text{If } ax^2 + bx + c = 0, \quad \text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

For example, suppose  $3x^2 + 7x - 6 = 0$ . Here  $a = 3$ ,  $b = 7$ , and  $c = -6$ . Substituting these values into the formula results in:

$$x = \frac{-7 \pm \sqrt{7^2 - 4(3)(-6)}}{2(3)} \Rightarrow x = \frac{-7 \pm \sqrt{121}}{6} \Rightarrow x = \frac{-7 \pm 11}{6}$$

Remember that non-negative numbers have both a positive and negative square root.

The sign  $\pm$  represents this fact for the square root in the formula and allows us to write the equation once (representing two possible solutions) until later in the solution process.

Split the numerator into the two values:  $x = \frac{-7 + 11}{6}$  or  $x = \frac{-7 - 11}{6}$

Thus the solution for the quadratic equation is:  $x = \frac{2}{3}$  or  $-3$ .

### Example 1

Solve:  $x^2 + 7x + 5 = 0$

First make sure the equation is in standard form with zero on one side of the equation. This equation is already in standard form.

Second, list the numerical values of the coefficients  $a$ ,  $b$ , and  $c$ . Since  $ax^2 + bx + c = 0$ , then  $a = 1$ ,  $b = 7$ , and  $c = 5$  for the equation  $x^2 + 7x + 5 = 0$ .

Write out the quadratic formula (see above). Substitute the numerical values of the coefficients  $a$ ,  $b$ , and  $c$  in the quadratic

formula,  $x = \frac{-7 \pm \sqrt{7^2 - 4(1)(5)}}{2(1)}$ .

Simplify to get the exact solutions.

$$x = \frac{-7 \pm \sqrt{49 - 20}}{2} \Rightarrow x = \frac{-7 \pm \sqrt{29}}{2},$$

$$\text{so } x = \frac{-7 + \sqrt{29}}{2} \quad \text{or} \quad \frac{-7 - \sqrt{29}}{2}$$

Use a calculator to get approximate solutions.

$$x \approx \frac{-7 + 5.39}{2} \approx \frac{-1.61}{2} \approx -0.81$$

$$x \approx \frac{-7 - 5.39}{2} \approx \frac{-12.39}{2} \approx -6.20$$

### Example 2

Solve:  $6x^2 + 1 = 8x$

First make sure the equation is in standard form with zero on one side of the equation.

$$\begin{array}{r} 6x^2 + 1 = 8x \\ -8x \quad -8x \\ \hline 6x^2 - 8x + 1 = 0 \end{array}$$

Second, list the numerical values of the coefficients  $a$ ,  $b$ , and  $c$ :  $a = 6$ ,  $b = -8$ , and  $c = 1$  for this equation.

Write out the quadratic formula, then substitute the values in the formula.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(6)(1)}}{2(6)}$$

Simplify to get the exact solutions.

$$x = \frac{8 \pm \sqrt{64 - 24}}{12} \Rightarrow x = \frac{8 \pm \sqrt{40}}{12} \Rightarrow \frac{8 \pm 2\sqrt{10}}{12}$$

$$\text{so } x = \frac{4 + \sqrt{10}}{6} \quad \text{or} \quad \frac{4 - \sqrt{10}}{6}$$

Use a calculator with the original answer to get approximate solutions.

$$x \approx \frac{8 + 6.32}{12} \approx \frac{14.32}{12} \approx 1.19$$

$$x \approx \frac{8 - 6.32}{12} \approx \frac{1.68}{12} \approx 0.14$$

Use the quadratic formula to solve the following equations.

1.  $x^2 + 8x + 6 = 0$

2.  $x^2 + 6x + 4 = 0$

3.  $x^2 - 2x - 30 = 0$

4.  $x^2 - 5x - 2 = 0$

5.  $7 = 13x - x^2$

6.  $15x - x^2 = 5$

7.  $x^2 = -14x - 12$

8.  $6x = x^2 + 3$

9.  $3x^2 + 10x + 5 = 0$

10.  $2x^2 + 8x + 5 = 0$

11.  $5x^2 + 5x - 7 = 0$

12.  $6x^2 - 2x - 3 = 0$

13.  $2x^2 + 9x = -1$

14.  $-6x + 6x^2 = 8$

15.  $3x - 12 = -4x^2$

16.  $10x^2 + 2x = 7$

17.  $2x^2 - 11 = 0$

18.  $3x^2 - 6 = 0$

19.  $3x^2 + 0.75x - 1.5 = 0$

20.  $0.1x^2 + 5x + 2.6 = 0$

## Answers

1.  $x \approx -0.84$  and  $-7.16$

2.  $x \approx -0.76$  and  $-5.24$

3.  $x \approx 6.57$  and  $-4.57$

4.  $x \approx 5.37$  and  $-0.37$

5.  $x \approx 12.44$  and  $0.56$

6.  $x \approx 14.66$  and  $0.34$

7.  $x \approx -0.92$  and  $-13.08$

8.  $x \approx 5.45$  and  $0.55$

9.  $x \approx -0.61$  and  $-2.72$

10.  $x \approx -0.78$  and  $-3.22$

11.  $x \approx 0.78$  and  $-1.78$

12.  $x \approx 0.89$  and  $-0.56$

13.  $x \approx -0.11$  and  $-4.39$

14.  $x \approx 1.76$  and  $-0.76$

15.  $x \approx 1.40$  and  $-2.15$

16.  $x \approx 0.74$  and  $-0.94$

17.  $x \approx -2.35$  and  $2.35$

18.  $x \approx -1.41$  and  $1.41$

19.  $x \approx 0.59$  and  $-0.84$

20.  $x \approx -0.53$  and  $-49.47$