

## SIMPLIFYING RATIONAL EXPRESSIONS

#21

**Rational expressions** are fractions that have algebraic expressions in their numerators and/or denominators. To simplify rational expressions find **factors** in the numerator and denominator that are the same and then write them as fractions equal to 1. For example,

$$\frac{6}{6} = 1 \quad \frac{x^2}{x^2} = 1 \quad \frac{(x+2)}{(x+2)} = 1 \quad \frac{(3x-2)}{(3x-2)} = 1$$

Notice that the last two examples involved binomial sums and differences. **Only** when sums or differences are **exactly** the same does the fraction equal 1. Rational expressions such as the examples below **cannot** be simplified:

$$\frac{(6+5)}{6} \quad \frac{x^3+y}{x^3} \quad \frac{x}{x+2} \quad \frac{3x-2}{2}$$

Most problems that involve rational expressions will require that you **factor** the numerator and denominator. For example:

$$\frac{12}{54} = \frac{2 \cdot 2 \cdot 3}{2 \cdot 3 \cdot 3 \cdot 3} = \frac{2}{9} \quad \text{Notice that } \frac{2}{2} \text{ and } \frac{3}{3} \text{ each equal 1.}$$

$$\frac{6x^3y^2}{15x^2y^4} = \frac{2 \cdot 3 \cdot x^2 \cdot x \cdot y^2}{5 \cdot 3 \cdot x^2 \cdot y^2 \cdot y^2} = \frac{2x}{5y^2} \quad \text{Notice that } \frac{3}{3}, \frac{x^2}{x^2}, \text{ and } \frac{y^2}{y^2} = 1.$$

$$\frac{x^2 - x - 6}{x^2 - 5x + 6} = \frac{(x+2)(x-3)}{(x-2)(x-3)} = \frac{x+2}{x-2} \quad \text{where } \frac{x-3}{x-3} = 1.$$

All three examples demonstrate that **all parts** of the numerator and denominator--whether constants, monomials, binomials, or factorable trinomials--must be written as products **before** you can look for factors that equal 1.

One special situation is shown in the following examples:

$$\frac{-2}{2} = -1 \quad \frac{-x}{x} = -1 \quad \frac{-x-2}{x+2} = \frac{-(x+2)}{x+2} = -1 \quad \frac{5-x}{x-5} = \frac{-(x-5)}{x-5} = -1$$

Note that in all cases we assume the denominator does not equal zero.

### Example 1

Simplify:  $\frac{(a^3b^{-2})^2}{a^4}$

Rewrite the numerator and denominator without negative exponents and parentheses.

$$\frac{(a^3b^{-2})^2}{a^4} \Rightarrow \frac{a^6b^{-4}}{a^4} \Rightarrow \frac{a^6}{a^4b^4}$$

Then look for the same pairs of factors that equal one (1) when divided. Writing out all of the factors can be helpful.

$$\frac{a \cdot a \cdot a \cdot a \cdot a \cdot a}{a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b} = 1 \cdot 1 \cdot 1 \cdot 1 \cdot \frac{a \cdot a}{b \cdot b \cdot b \cdot b}$$

Write the simplified expression with exponents.

$$\frac{(a^3b^{-2})^2}{a^4} = \frac{a^2}{b^4}, b \neq 0. \text{ Note that } \frac{a}{a} = 1.$$

### Example 2

Simplify:  $\frac{2x^2-13x-7}{x^2-4x-21}$

To simplify some rational expressions, the numerator and/or denominator may need to be factored before you may simplify the expression.

$$\frac{2x^2-13x-7}{x^2-4x-21} \Rightarrow \frac{(2x+1)(x-7)}{(x-7)(x+3)}$$

Then look for the same pairs of factors that equal one (1) when divided.

$$\frac{(2x+1)(x-7)}{(x+3)(x-7)} \Rightarrow \frac{2x+1}{x+3} \cdot 1 \Rightarrow \frac{2x+1}{x+3} \text{ for } x \neq -3 \text{ or } 7.$$

Note that  $\frac{(x-7)}{(x-7)} = 1$ .

Simplify the following expressions. Assume that the denominator is not equal to zero.

- |                                   |  |                                      |  |
|-----------------------------------|--|--------------------------------------|--|
| 1. $\frac{12x^2y^4}{3x^2y^3}$     | 2. $\frac{10a^6b^8}{40a^2b^2}$         | 3. $\frac{(x^5y^3)^3}{x^{12}y}$      | 4. $\frac{(a^5)^2}{a^{13}b^6}$         |
| 5. $\frac{(5x^3)^2y^3}{10xy^9}$   | 6. $\frac{3(a^3)^5b}{(3a^4)^3b^{10}}$  | 7. $\frac{4ab^{-5}}{a^8b}$           | 8. $\frac{2x^{-3}y^8}{4x^{-2}}$        |
| 9. $\frac{(x^8y^{-3})^{-2}}{x^2}$ | 10. $\frac{2x^3y^{-1}}{6(4x)^{-2}y^7}$ | 11. $\frac{(2x-1)(x+3)}{(x-5)(x+3)}$ | 12. $\frac{(5x-1)(x+2)}{(x+7)(5x-1)}$  |
| 13. $\frac{3x+1}{3x^2+10x+3}$     | 14. $\frac{x^2-x-20}{x-5}$             | 15. $\frac{3x-6}{x^2+4x-12}$         | 16. $\frac{2x^2-x-3}{10x-15}$          |
| 17. $\frac{3x^2+x-10}{x^2+6x+8}$  | 18. $\frac{x^2-64}{x^2+16x+64}$        | 19. $\frac{4x^2-x}{4x^3+11x^2-3x}$   | 20. $\frac{2x^3+2x^2-12x}{8x^2-8x-16}$ |

### Answers

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|--|--|---|
| 1. $4y$  | 2. $\frac{a^4b^6}{4}$  | 3. $\frac{x^{15}y^9}{x^{12}y} = x^3y^8$                 |
| 4. $\frac{a^{10}}{a^{13}b^6} = \frac{1}{a^3b^6}$   | 5. $\frac{25x^6y^3}{10xy^9} = \frac{5x^5}{2y^6}$                                     | 6. $\frac{3a^{15}b}{27a^{12}b^{10}} = \frac{a^3}{9b^9}$ |
| 7. $\frac{4a}{a^8b^6} = \frac{4}{a^7b^6}$          | 8. $\frac{2x^2y^8}{4x^3} = \frac{y^8}{2x}$   | 9. $\frac{x^{-16}y^6}{x^2} = \frac{y^6}{x^{18}}$        |
| 10. $\frac{32x^5}{6y^8} = \frac{16x^5}{3y^8}$      | 11. $\frac{2x-1}{x-5}$   | 12. $\frac{x+2}{x+7}$                                   |
| 13. $\frac{3x+1}{(x+3)(3x+1)} = \frac{1}{x+3}$     | 14. $\frac{(x-5)(x+4)}{x-5} = x+4$   | 15. $\frac{3(x-2)}{(x-2)(x+6)} = \frac{3}{x+6}$         |
| 16. $\frac{(2x-3)(x+1)}{5(2x-3)} = \frac{x+1}{5}$  | 17. $\frac{(3x-5)(x+2)}{(x+4)(x+2)} = \frac{3x-5}{x+4}$                              | 18. $\frac{(x+8)(x-8)}{(x+8)(x+8)} = \frac{x-8}{x+8}$   |
| 19. $\frac{x(4x-1)}{x(4x-1)(x+3)} = \frac{1}{x+3}$ | 20. $\frac{2x(x+3)(x-2)}{8(x-2)(x+1)} = \frac{x(x+3)}{4(x+1)} = \frac{x^2+3x}{4x+4}$ |   |