

MULTIPLICATION AND DIVISION OF RATIONAL EXPRESSIONS

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To multiply or divide rational expressions, follow the same procedures used with numerical fractions. However, it is often necessary to factor the polynomials in order to simplify the rational expression.

Example 1

Multiply $\frac{x^2 + 6x}{(x+6)^2} \cdot \frac{x^2 + 7x + 6}{x^2 - 1}$ and simplify the result.

After factoring, the expression becomes:

$$\frac{x(x+6)}{(x+6)(x+6)} \cdot \frac{(x+6)(x+1)}{(x+1)(x-1)}$$

After multiplying, reorder the factors:

$$\frac{(x+6)}{(x+6)} \cdot \frac{(x+6)}{(x+6)} \cdot \frac{x}{(x-1)} \cdot \frac{(x+1)}{(x+1)}$$

Since $\frac{(x+6)}{(x+6)} = 1$ and $\frac{(x+1)}{(x+1)} = 1$, simplify:

$$1 \cdot 1 \cdot \frac{x}{x-1} \cdot 1 \Rightarrow \frac{x}{x-1}$$

Note: $x \neq -6, -1, \text{ or } 1$.

Example 2

Divide $\frac{x^2 - 4x - 5}{x^2 - 4x + 4} \div \frac{x^2 - 2x - 15}{x^2 + 4x - 12}$ and simplify the result.

First, change to a multiplication expression by inverting (flipping) the second fraction:

$$\frac{x^2 - 4x - 5}{x^2 - 4x + 4} \cdot \frac{x^2 + 4x - 12}{x^2 - 2x - 15}$$

After factoring, the expression is:

$$\frac{(x-5)(x+1)}{(x-2)(x-2)} \cdot \frac{(x+6)(x-2)}{(x-5)(x+3)}$$

Reorder the factors (if you need to):

$$\frac{(x-5)}{(x-5)} \cdot \frac{(x-2)}{(x-2)} \cdot \frac{(x+1)}{(x-2)} \cdot \frac{(x+6)}{(x+3)}$$

Since $\frac{(x-5)}{(x-5)} = 1$ and $\frac{(x-2)}{(x-2)} = 1$, simplify:

$$\frac{(x+1)(x+6)}{(x-2)(x+3)}$$

Thus, $\frac{x^2 - 4x - 5}{x^2 - 4x + 4} \div \frac{x^2 - 2x - 15}{x^2 + 4x - 12} = \frac{(x+1)(x+6)}{(x-2)(x+3)}$ or $\frac{x^2 + 7x + 6}{x^2 + x - 6}$. Note: $x \neq -3, 2, \text{ or } 5$.

Multiply or divide each expression below and simplify the result. Assume the denominator is not equal to zero.

1. $\frac{3x+6}{5x} \cdot \frac{x+4}{x^2+2x}$

2. $\frac{8a}{a^2-16} \cdot \frac{a+4}{4}$

3. $\frac{x^2-1}{3} \cdot \frac{2}{x^2-x}$

4. $\frac{x^2-x-12}{x^2} \cdot \frac{x}{x-4}$

5. $\frac{x^2-16}{(x-4)^2} \cdot \frac{x^2-3x-18}{x^2-2x-24}$

6. $\frac{x^2+6x+8}{x^2-4x+3} \cdot \frac{x^2-5x+4}{5x+10}$

7. $\frac{x^2-x-6}{x^2-x-20} \cdot \frac{x^2+6x+8}{x^2-x-6}$
8. $\frac{x^2-x-30}{x^2+13x+40} \cdot \frac{x^2+11x+24}{x^2-9x+18}$
9. $\frac{3x+12}{x^2} \div \frac{x+4}{x}$
10. $\frac{2a+6}{a^3} \div \frac{a+3}{a}$
11. $\frac{15-5x}{x^2-x-6} \div \frac{5x}{x^2+6x+8}$
12. $\frac{17x+119}{x^2+5x-14} \div \frac{9x-1}{x^2-3x+2}$
13. $\frac{x^2+8x}{9x} \div \frac{x^2-64}{3x^2}$
14. $\frac{x^2-1}{x^2-6x-7} \div \frac{x^3+x^2-2x}{x-7}$
15. $\frac{2x^2-5x-3}{3x^2-10x+3} \div \frac{4x^2+4x+1}{9x^2-1}$
16. $\frac{x^2+3x-10}{x^2+3x} \div \frac{x^2-4x+4}{4x+12}$
17. $\frac{x^2-x-6}{x^2+3x-10} \cdot \frac{x^2+2x-15}{x^2-6x+9} \cdot \frac{x^2+4x-21}{x^2+9x+14}$
18. $\frac{3x^2+21x}{x^2-49} \cdot \frac{x^2-x}{6x^3-9x^2} \cdot \frac{4x^2-9}{3x-3}$
19. $\frac{4x^3+7x-2x}{2x^2-162} \div \frac{4x^2+15x-4}{12x-60} \cdot \frac{x^2+9x}{x^2-3x-10}$
20. $\frac{10x^2-11x+3}{x^2-6x-40} \cdot \frac{x^2+11x+28}{2x^2-x} \div \frac{x+7}{2x^2-20x}$

Answers

1. $\frac{3(x+4)}{5x^2} = \frac{3x+12}{5x^2}$
2. $\frac{2a}{a-4}$
3. $\frac{2(x+1)}{3x} = \frac{2x+2}{3x}$
4. $\frac{x+3}{x}$
5. $\frac{(x+3)}{(x-4)}$
6. $\frac{(x+4)(x-4)}{5(x-3)} = \frac{x^2-16}{5x-15}$
7. $\frac{(x+2)}{(x-5)}$
8. $\frac{(x+3)}{(x-3)}$
9. $\frac{3}{x}$
10. $\frac{2}{a^2}$
11. $\frac{-(x+4)}{x} = \frac{-x-4}{x}$
12. $\frac{17(x-1)}{9x-1} = \frac{17x-17}{9x-1}$
13. $\frac{x^2}{3(x-8)} = \frac{x^2}{3x-24}$
14. $\frac{1}{x(x+2)}$
15. $\frac{(3x+1)}{(2x+1)}$
16. $\frac{4(x+5)}{x(x-2)} = \frac{4x+20}{x^2-2x}$
17. $\frac{(x-3)}{(x-2)}$
18. $\frac{2x+3}{3(x-7)} = \frac{2x+3}{3x-21}$
19. $\frac{6x^2}{(x-9)(x+4)} = \frac{6x^2}{x^2-5x-36}$
20. $\frac{2(5x-3)}{1} = 10x-6$