

ABSOLUTE VALUE EQUATIONS

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Absolute value means the distance from a reference point. In the simplest case, the absolute value of a number is its distance from zero on the number line. Since absolute value is a distance, the result of finding an absolute value is zero or a positive number. All distances are positive.

Example 1

$$\text{Solve } |2x + 3| = 7.$$

Because the result of $(2x + 3)$ can be 7 or -7, we can write and solve two different equations. (Remember that the absolute value of 7 and -7 will be 7.)

$$2x + 3 = 7 \text{ or } 2x + 3 = -7$$

$$2x = 4 \text{ or } 2x = -10$$

$$x = 2 \text{ or } x = -5$$

Example 2

$$\text{Solve } 2|2x + 13| = 10.$$

First the equation must have the absolute value isolated on one side of the equation.

$$2|2x + 13| = 10 \Rightarrow |2x + 13| = 5$$

Because the result of $2x + 13$ can be 5 or -5, we can write and solve two different equations.

$$2x + 13 = 5 \text{ or } 2x + 13 = -5$$

$$2x = -8 \text{ or } 2x = -18$$

$$x = -4 \text{ or } x = -9$$

Note that while some x-values of the solution are negative, the goal is to find values that make the original absolute value statement true. For $x = -5$ in example 1, $|2(-5) + 3| = 7 \Rightarrow |-10 + 3| = 7 \Rightarrow |-7| = 7$, which is true. Verify that the two negative values of x in example 2 make the original absolute value equation true.

Solve for x .

1. $|x + 2| = 4$

2. $|3x| = 27$

3. $|x - 5| = 2$

4. $|x - 8| = 2$

5. $|\frac{x}{5}| = 2$

6. $|-3x| = 4$

7. $|3x + 4| = 10$

8. $|12x - 6| = 6$

9. $|x| + 3 = 20$

10. $|x| - 8 = -2$

11. $2|x| - 5 = 3$

12. $4|x| - 5 = 7$

13. $|x + 2| - 3 = 7$

14. $|x + 5| + 4 = 12$

15. $|2x - 3| + 2 = 11$

16. $-3|x| + 5 = -4$

17. $-3|x + 6| + 12 = 0$

18. $15 - |x + 1| = 3$

19. $14 + 2|3x + 5| = 26$

20. $4|x - 10| - 23 = 37$

Answers

1. $x = 2, -6$

2. $x = 9, -9$

3. $x = 7, 3$

4. $x = 10, 6$

5. $x = 10, -10$

6. $x = -\frac{4}{3}, \frac{4}{3}$

7. $x = 2, -\frac{14}{3}$

8. $x = 1, 0$

9. $x = 17, -17$

10. $x = 6, -6$

11. $x = 4, -4$

12. $x = 3, -3$

13. $x = 8, -12$

14. $x = 3, -13$

15. $x = 6, -3$

16. $x = 3, -3$

17. $x = -2, -10$

18. $x = 11, -13$

19. $x = \frac{1}{3}, -\frac{11}{3}$

20. $x = 25, -5$