

ADDITION AND SUBTRACTION OF RATIONAL EXPRESSIONS

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Addition and subtraction of rational expressions is done the same way as addition and subtraction of numerical fractions. Change to a common denominator (if necessary), combine the numerators, and then simplify.

Example 1

The Least Common Multiple (lowest common denominator) of $(x + 3)(x + 2)$ and $(x + 2)$ is $(x + 3)(x + 2)$.

The denominator of the first fraction already is the Least Common Multiple. To get a common denominator in the second fraction, multiply the fraction by $\frac{x+3}{x+3}$, a form of one (1).

Multiply the numerator and denominator of the second term:

Distribute in the second numerator.

Add, factor, and simplify. Note: $x \neq -2$ or -3 .

$$\frac{4}{(x+2)(x+3)} + \frac{2x}{x+2}$$

$$= \frac{4}{(x+2)(x+3)} + \frac{2x}{x+2} \cdot \frac{(x+3)}{(x+3)}$$

$$= \frac{4}{(x+2)(x+3)} + \frac{2x(x+3)}{(x+2)(x+3)}$$

$$= \frac{4}{(x+2)(x+3)} + \frac{2x^2+6x}{(x+2)(x+3)}$$

$$= \frac{2x^2+6x+4}{(x+2)(x+3)} = \frac{2(x+1)(x+2)}{(x+2)(x+3)} = \frac{2(x+1)}{(x+3)}$$

Example 2

Subtract $\frac{3}{x-1} - \frac{2}{x-2}$ and simplify the result.

Find the lowest common denominator of $(x - 1)$ and $(x - 2)$. It is $(x - 1)(x - 2)$.

In order to change each denominator into the lowest common denominator, we need to multiply each fraction by factors that are equal to one.

Multiply the denominators.

Multiply and distribute the numerators.

When adding fractions, the denominator does not change. The numerators need to be added or subtracted and like terms combined.

Check that both the numerator and denominator are completely factored. If the answer can be simplified, simplify it. This answer is already simplified. Note: $x \neq 1$ or 2 .

$$\frac{(x-2)}{(x-2)} \cdot \frac{3}{x-1} - \frac{2}{(x-2)} \cdot \frac{(x-1)}{(x-1)}$$

$$\frac{3(x-2)}{(x-2)(x-1)} - \frac{2(x-1)}{(x-2)(x-1)}$$

$$\frac{3x-6}{(x-2)(x-1)} - \frac{2x-2}{(x-2)(x-1)}$$

$$\frac{3x-6-(2x-2)}{(x-2)(x-1)} \Rightarrow \frac{3x-6-2x+2}{(x-2)(x-1)} \Rightarrow \frac{x-4}{(x-2)(x-1)}$$

$$\frac{x-4}{(x-2)(x-1)} = \frac{x-4}{x^2-3x+2}$$

Add or subtract the expressions and simplify the result.

$$1. \frac{x}{(x+2)(x+3)} + \frac{2}{(x+2)(x+3)}$$

$$2. \frac{x}{x^2+6x+8} + \frac{4}{x^2+6x+8}$$

$$3. \frac{b^2}{b^2+2b-3} + \frac{-9}{b^2+2b-3}$$

$$4. \frac{2a}{a^2+2a+1} + \frac{2}{a^2+2a+1}$$

$$5. \frac{x+10}{x+2} + \frac{x-6}{x+2}$$

$$6. \frac{a+2b}{a+b} + \frac{2a+b}{a+b}$$

$$7. \frac{3x-4}{3x+3} - \frac{2x-5}{3x+3}$$

$$8. \frac{3x}{4x-12} - \frac{9}{4x-12}$$

$$9. \frac{6a}{5a^2+a} - \frac{a-1}{5a^2+a}$$

$$10. \frac{x^2+3x-5}{10} - \frac{x^2-2x+10}{10}$$

$$11. \frac{6}{x(x+3)} + \frac{2}{x+3}$$

$$12. \frac{5}{x-7} + \frac{3}{4(x-7)}$$

$$13. \frac{5x+6}{x^2} - \frac{5}{x}$$

$$14. \frac{2}{x+4} - \frac{x-4}{x^2-16}$$

$$15. \frac{10a}{a^2+6a} - \frac{3}{3a+18}$$

$$16. \frac{3x}{2x^2-8x} + \frac{2}{(x-4)}$$

$$17. \frac{5x+9}{x^2-2x-3} + \frac{6}{x^2-7x+12}$$

$$18. \frac{x+4}{x^2-3x-28} - \frac{x-5}{x^2+2x-35}$$

$$19. \frac{3x+1}{x^2-16} - \frac{3x+5}{x^2+8x+16}$$

$$20. \frac{7x-1}{x^2-2x-3} - \frac{6x}{x^2-x-2}$$

Answers

$$1. \frac{1}{x+3}$$

$$2. \frac{1}{x+2}$$

$$3. \frac{b-3}{b-1}$$

$$4. \frac{2}{a+1}$$

$$5. 2$$

$$6. 3$$

$$7. \frac{1}{3}$$

$$8. \frac{3}{4}$$

$$9. \frac{1}{a}$$

$$10. \frac{x-3}{2}$$

$$11. \frac{2}{x}$$

$$12. \frac{23}{4(x-7)} = \frac{23}{4x-28}$$

$$13. \frac{6}{x^2}$$

$$14. \frac{1}{x+4}$$

$$15. \frac{9}{(a+6)}$$

$$16. \frac{7}{2(x-4)} = \frac{7}{2x-8}$$

$$17. \frac{5(x+2)}{(x-4)(x+1)} = \frac{5x+10}{x^2-3x-4}$$

$$18. \frac{14}{(x+7)(x-7)} = \frac{14}{x^2-49}$$

$$19. \frac{4(5x+6)}{(x-4)(x+4)^2}$$

$$20. \frac{x+2}{(x-3)(x-2)} = \frac{x+2}{x^2-5x+6}$$